

The Beginnings of Spontaneous Symmetry Breaking in Particle Physics

— Derived From My on the Spot “Intellectual Battlefield Impressions”

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I summarize the development of the ideas of Spontaneous symmetry breaking in the 1960s with an outline of the Guralnik, Hagen, Kibble (GHK) paper and include comments on the relationship of this paper to those of Brout, Englert and Higgs. I include some pictures from my “physics family album which might be of some amusement value”

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1. Introduction

This paper borrows freely from my review in IJMPA [1] as well as previous talks given by Hagen, Kibble and myself. I give an overview of the status of particle physics in the 1960s with particular focus on the work that I did with Richard Hagen and Tom Kibble (GHK) [4]. Our group and two others, Englert-Brout (EB) [5] and Higgs (H) [6, 7] worked on what is now commonly referred to as the “Higgs” phenomenon and/or a specific example of this constructed through the spontaneously broken scalar electrodynamics. This work eventually led to the unified theory of electroweak interaction as developed by Weinberg and Salam [8, 9]. I discuss how I understand symmetry breaking. My overall viewpoint has changed little in basics over the years. However, with the hope of clarifying this viewpoint, I mention some more modern ideas which involve the understanding of the full range of solutions of quantum field theories [2, 3]. Later in this document, in response to statements that have been made in print and at other conferences, I discuss the differences between our work and that of EB and H.

2. What was theoretical particle physics like in the early 1960s?

It is ironic that just before the “Golden age” of particle physics began to unfold in the 1960s, Quantum Field theory was widely thought to have reached a dead end. There was no understanding as to how to go beyond coupling constant perturbation and thus there seemed to be little hope of performing detailed calculations of strong interactions. As a result but for a few places, including Harvard, MIT and Imperial college (the GHK home institutions), most theoretical research attention was focussed on S Matrix theory.

Fortunately, with the notable exception of lattice computational methods, the basic field theory tools and formulations we use today were, for the most part, available in some form. The Schwinger Action Principle with associated Green’s Function methods were well developed, but mostly used by physicists who were in Schwinger’s Harvard centered following. The (equivalent) Feynman Path Integral path integral method also existed although its application was probably even less known than Schwinger’s methods. That either of these methods could easily lead to solutions of quantum field theory that were fundamentally non (coupling constant) perturbative was, at the beginning of the 60s, at most an idea in waiting. This idea was soon to be developed with the introduction of the concept of spontaneous symmetry breaking. Progress still continues as illustrated by the fact that only recently was it understood that path integrals must include paths extended into the complex plane for them to encompass the extended range of solutions to quantum field theory [2, 3]. Coupling constant perturbation theory was well understood and widely applied as the major computation tool of quantum field theory. As an essential adjunct, renormalization theory was well developed and had enjoyed sensational success in application to problems involving fermions interacting electromagnetically. However, it is of relevance to note that many physicists were very unclear as to how to work with operators in quantum electrodynamics as was evidenced by a fairly constant stream of papers using cumbersome indefinite metric formulations. This was

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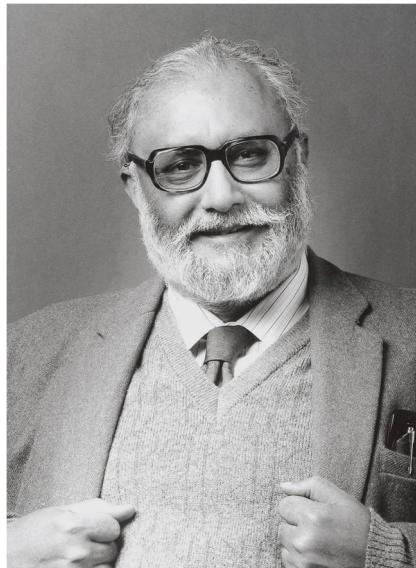
not an issue if Schwinger's methods of quantizing in the radiation gauge were followed. In the end, of course, everyone ended up calculating the same graphs but on the way to this there were many ways to make errors of interpretations.

There was deep concern about the four-fermion interaction not being renormalizable since this cast serious doubt on our understanding of weak interactions. This was particularly frustrating since the lowest order perturbative calculations using four fermion interactions described many weak processes with surprising accuracy. There was some experimentation with summation techniques of subsets of graphs with hope of understanding this problem.

Very importantly, we had the beginnings of the Standard Model. The idea of electromagnetic like interactions of multiplets of fields was introduced with the ideas of Yang, Mills and Salam's student Shaw [13, 14] in 1954. J.J Sakurai [15] had suggested the fundamental importance of vector interactions to a basic model of particle interaction.

Essential to putting this all together was the contribution of the V-A theory of the weak interactions by Sudarshan and Marshak (1957) which was according to Feynman in 1963 "publicized by Feynman and Gell-Mann" [21]. The realization that the V-A combination is the valid way to describe the weak interactions is essential to the unified Electroweak theory and the standard model.

The importance of group theory to categorizing the elementary particles was approximate flavor $SU(3)$ was well appreciated due to the work of Murray Gell-Mann [16] and Y. Ne'mann [17] (another Salam Student) and regarded as largely confirmed thanks to the discovery in 1963 of the Ω^- which was needed to fill in the baryon decuplet (10 particles).



Abdus Salam

The Gell-Mann–Zweig quark (aces) ideas were formulated by 1964 [18, 19] but were far from completely accepted. The idea of fundamental quarks became theoretically feasible due to the 1963 work by Wally Greenberg [20]. His ideas lead to the concept of color and made provided a workable way to construct fermions out of 3 quarks.

The story I want to focus on began in 1961 when Nambu with Jona-Lasinio (NJL) published [22, 23] their study of spontaneous symmetry breaking of chiral symmetry using the four fermion interaction

$$g [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2].$$

This interaction in itself was disturbing (just as was the four fermion interaction to describe weak processes) because perturbation theory with expansion around $g = 0$ produces a series of increasingly divergent terms that cannot be renormalized. NJL studied this model by imposing a constraint that, at first glance, seemed to be inconsistent with its symmetry and then formulated a new (not coupling constant perturbation theory) leading order approximation. This formulation led to a constraint that allowed consistency if a zero mass pseudoscalar boson appeared in the calculated particle spectrum. This is the first example of the Nambu- Goldstone boson. NJL argued, without an entirely convincing proof, that the massless particle is an exact requirement following from the imposition of dynamical symmetry breaking. This was a truly major breakthrough with new physics

and a new way at looking at solutions of relativistic quantum field theory! We understood little beyond coupling constant perturbation theory about how to actually solve a QFT. Attempts had been made to re-sum these series but until NJL there was little or no realization that one should be looking for an entirely different solutions and that these solutions could violate obvious symmetries of the action.

After the NJL papers, J. Goldstone wrote (1961) [24] his famous paper where he examined a two (real) component scalar field theory with a quartic self interaction. This presented a much simpler illustration of a spontaneous symmetry breaking than that of NJL. Here the interaction has a conserved charge symmetry which is dynamically broken by requiring that the vacuum expectation of the scalar fields non-zero. In this case, the leading order approximation to the broken symmetric solution can be understood without dealing with constrained divergent integrals. A constraint, of course, is still present but it occurs as a simple numerical relation (to leading order). While the original Lagrangian contains two equal mass scalar fields, the resulting solutions (again and importantly to leading order) look like those of a free field with two scalar bosons of different mass with one mass vanishing and the square magnitude of the other mass turning out to be twice the magnitude of the original “bare mass”. Goldstone speculates that the zero mass excitation is required to all orders. In fact, higher order calculations result in graphical structures based on the leading order propagators and the massless particle stay massless while the mass of the other particle must be renormalized and, of course, varies order by order.

In 1962 Goldstone Salam and Weinberg [25] convincingly proved that the spontaneous breaking of a continuous global symmetry in a relativistic theory requires associated zero mass excitations. This was a disappointment as they had hoped to find that there was a way around this connection. Zero mass particles in nature are simply not seen with the exception of the photon and presumably the graviton. Consequently, the idea of spontaneous symmetry breaking seemed like it could not lead to new physics. Describing how this apparent problem was solved is the main point of this paper. But before I get to this I will indicate why the whole appearance of additional solutions to Quantum Field theory (QFT) beyond those given by perturbation theory are to be expected and are in fact suggested (and explained) by the solution set of ordinary differential differential equations. This analysis is relatively new, but I think helps to clarify what is happening here even for those without much detail knowledge of the mathematics of QFT.

The beginning point to understand the multiplicity of solutions to QFT is the 1952 work of Dyson [26]. This paper showed that there is a singularity at zero coupling in QED, thus giving an early indicator that perturbation theory could not be the whole story and indeed that the perturbation expansion is asymptotic not convergent. As mentioned above, we can get insight by examining simple differential equations. As a starter, it is not hard to see, using the Feynman path integral or the Schwinger action principle, that such equations can be interpreted as describing a zero dimensional QFT. As an explicit example, consider,

$$g \frac{d^3y}{dJ^3} + m^2 \frac{dy}{dJ} = J .$$

How many solutions does this have? Three! They can be found from an integral representation (zero dimensional Feynman path integral for quartic interacting scalar field theory):

$$Z = \int e^{-g \frac{\phi^4}{4} - \frac{m^2}{2} \phi^2 + J \phi} \mathcal{D}\phi$$

The J derivative of Z satisfies the original differential equation when the integral is evaluated over COMPLEX paths which do not contribute at the end points. It is easy to show that there are 3 allowed independent paths in the complex plane. Associated with the three integration paths, the integral has 3 stationary points that correspond to the three solutions of the original differential equation.

These are ($J = 0$) located at

$$\phi = 0, \phi = \pm \sqrt{\frac{-m^2}{g}} .$$

Expanding around these stationary phase points to discover asymptotic expressions for each of the three solutions is straightforwar. The path along the real axis corresponds to the stationary point at $\phi = 0$ and is the familiar solution found by perturbation expansion around $g = 0$. The perturbative solution vanishes at $J = 0$ is regular in g at $g = 0$. The other solutions break reflection symmetry and are singular at $g = 0$. The symmetry breaking solutions always show singularities as the coupling vanishes. You often have to be very careful to not miss these.

Now returning to higher dimensions (which, in principle, can be constructed from zero dimensional solutions) a new element occurs - namely the Nambu Goldstone Theorem. This (roughly) states that in more than 2 spacetime dimensions) that if a *charge* associated with a *conserved current* in a relativistic field theory does not *destroy* the vacuum \Rightarrow the theory has zero mass excitations. At first glance the theorem seems to be true using exact results of QFT without use of *any* approximation techniques. Furthermore, in an interacting theory, you can not break a symmetry using a coupling constant based perturbative expansion around zero coupling strength.

What is Goldstone’s Theorem good for? As mentioned above, before 1964 it appeared to be an impediment to creatively use symmetry breaking because of the dearth of physical massless particles.

This is where I come in: After Bjorken gave a talk (1962) at Harvard, my thesis advisor, Walter Gilbert (Nobel Laureate Chemistry 1980), suggested that I look at Bjorken’s proposed model of E&M [28] — a variant of the Nambu–Jona-Lasinio model with interaction

$$g (\bar{\psi} \gamma^\mu \psi) (\bar{\psi} \gamma_\mu \psi) .$$

The current is required to have non-vanishing vacuum expectation. The symmetry that is broken is Lorentz symmetry — relativistic invariance.

In my Ph.D thesis, I showed that BJ’s basic conclusion that this theory is equivalent to QED is correct. Careful calculation shows that the Lorentz symmetry breaking is trivial and does not manifest itself in a physically observable way [11, 12]. This is a surprise since in *coupling constant perturbation theory* this interaction leads to hopelessly divergent results. In fact, this calculation provides results in a different [quantum] phase corresponding to symmetry breaking boundary conditions and is an entirely different solution than the non-existent coupling constant perturbation expansion. This is the direct analog to the multiple solutions of ordinary differential equations previously discussed.

Despite the fact that by this time Schwinger [29] had argued that there was no dynamical reason for the photon to have zero mass, I thought from what I had learned about the Bjorken model that I could construct a symmetry breaking argument that disproved Schwinger’s conclusions and demonstrated that conventional E&M must, in general, have massless photons. I included this in my thesis. My argument was wrong and, fortunately, Coleman detected this in my (1963) thesis presentation. I removed the offending chapter in the final version.

Somewhat before my thesis was finished I had discussed a related project with Gilbert. He made the observation that the action of a massless *scalar* particle (B) and a massless *vector* particle (A^λ) with the simple “interaction”

$$g A^\lambda (\partial_\lambda B - g A_\lambda)$$

produces a free spin 1 field with mass g^2 . This can be *anticipated* by *counting* degrees of freedom and noting that g carries the dimension of mass (this model has a conserved current and a trace of gauge invariance).

I told David Boulware about this and he spoke to Gilbert and they wrote a paper including some additional discussion about massless limits of massive vector meson theories [30]. Thus, at this time, the 2-dimensional Schwinger model (E&M in 2 dimensions) showed that gauge theories need not have zero mass and the BG model in 4 dimensions *confirmed* this again. It is a *easy* step from the BG model to the lowest approximation used in the GHK paper. They are essentially the same! With hindsight, all the ingredients for the GHK paper were available at Harvard in 1962!

During my time at Harvard, I was talking with Dick Hagen an undergraduate friend at MIT and then a Physics graduate student at MIT and already my co-author on our first physics research paper [10].



A Working Afternoon: Young Baron May of Oxford, Guralnik and Hagen 1961

In 1963 Hagen took a postdoctoral position at the University of Rochester (and is still there). We continued collaborating. He became interested in complicated expensive and unreliable but beautiful machinery - see the photos below. Also he became an expert on how to minimize living costs. For a while, I thought he might be thinking of becoming a very “hands on” experimentalist.



I went to Imperial College (after being rejected by CERN) at the beginning of 1964 with a new NSF postdoctoral fellowship and the certainty that *something interesting* happened with gauge theories and *symmetry breaking*. IC was probably the best High Energy Theory place in the world at that time and I met a fantastic bunch of physicists there. The ones I interacted with the most were Tom Kibble, Ray, Streater, John Charap, and to a lesser degree Paul Matthews and Abdus Salam.



Salam and Kibble -like the Harvard and MIT crowd, the IC people were very serious

I quickly learned that while Harvard was relatively safe field theory ground, protected by Schwinger's large (but indifferent) umbrella, the idea that there was even such a thing as symmetry breaking in field theory was not universally accepted — even at IC where Salam with Goldstone and Weinberg had already published their nice paper on these ideas [25].

Ray Streater (an axiomatic or constructive field theorist) stated that his community did not believe that symmetry breaking was possible. A lot of arguing and careful construction of a free model *convinced* him that the axioms were too restrictive. Later, he published a paper on this [32], which amusingly got a lot more attention than the paper I published in PRL giving the simple free example and a significant example of the Goldstone theorem in gauge theory but with an incomplete analysis of the resulting gauge structure [27].

The understanding of my oversight in this paper (which, incidentally, was also caught by Dave Boulware) was the final key to the GHK understanding, within the context of the Goldstone theorem and without resorting to perturbation theory, why symmetry breaking in a gauge theory, does not require massless particles. In gauge theories the assumptions of the Goldstone theorem are easily (and always) violated in physical gauges, or equivalently, are only applicable to non-physical excitations.

I begin by re-examining my earlier PRL paper. As I proceed, it will be clear why my explanation is the basis of the GHK paper. The proof in QED is straightforward: There is an asymmetric conserved tensor current,

$$\begin{aligned} J^{\mu\nu} &= F^{\mu\nu} - x^\nu J^\mu \\ \partial_\mu J^{\mu\nu} &= 0 \\ \Rightarrow Q^\nu &= \int d^3x [F^{0\nu} - x^\nu J^0] \\ \text{and } \frac{d}{dt} Q^\nu &= 0. \end{aligned}$$

We use the gauge $\vec{\nabla} \cdot \vec{A} = 0$ (a very natural gauge in operator QED) so that we only deal with physical excitations.

By the commutation relations it is easily seen that this requires

$$\langle 0 | [Q^k, A^l(\vec{x}, t)] | 0 \rangle = (\text{non-zero constant}).$$

However, direct calculation using spectral representations show that this expression is time-dependent for $e \neq 0$!

What went wrong? The radiation gauge is not explicitly Lorentz invariant, and we cannot use *causality* arguments to prove that the *commutator* above is *confined* to a local region of space-time. This means that, even though $\partial_0 J^{00} + \partial_k J^{0k} = 0$, we cannot neglect surface integrals of J^{0k} . It follows that our weird **charge leaks out of any volume!** This leads us, at once, to re-consider the proof of Goldstone's theorem.

What have we learned?

Goldstone's theorem is true for a manifestly covariant theory, i.e., a theory where $\partial_\mu J^\mu = 0$ and *surface terms* vanish *fast enough* so that

$$\begin{aligned} \langle 0 | \left[\int d^3x (\partial_\mu J^\mu), (\text{local operator}) \right] | 0 \rangle &= \\ &= \frac{d}{dt} \langle 0 | \left[\int d^3x J^0, (\text{local operator}) \right] | 0 \rangle . \end{aligned}$$

That is to say:

$$Q = \int d^3x J^0$$

has a zero mass particle in its spectrum. This includes electromagnetism with the special charge introduced above if you re-gauge to a manifestly covariant gauge. However, in this case, you can demonstrate exactly that the zero mass particles are gauge excitations. Note that these are very general statements: Goldstone's theorem need not require physical zero mass states in any gauge theory (and it does not). This is because these theories are made to be relativistic by introducing extra gauge degrees of freedom. Indeed, the Goldstone bosons are always nonphysical.

There is no reason for the photon to be massless in normal QED, but the smallness of the coupling constant and hence the applicability of perturbation theory. We can see an approximate example of the failure of Goldstone's theorem by looking at the action

$$\begin{aligned} L = & -\frac{1}{2} F^{\mu\nu} (\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \phi^\mu \partial_\mu \phi + \\ & + \frac{1}{2} \phi^\mu \phi_\mu + i e_0 \phi^\mu \mathbf{q} \phi A_\mu \\ \mathbf{q} &= \sigma_2 \\ \phi &= (\phi_1, \phi_2) \\ \phi_\mu &= (\phi_1^\mu, \phi_2^\mu) \end{aligned}$$

This is the Lagrangian for scalar electrodynamics. It is a very non-trivial interacting theory characterized by a conserved current. It is renormalizable in the coupling constant expansion with an induced ϕ^4 interaction. No other non-trivial ϕ^n interaction can be added to it and keep it renormalizable. Here and in our paper we use the convention followed at Harvard and MIT by the “Schwinger School” and do not introduce (unneeded) explicit counter-terms. This property and much of the general behavior of scalar electrodynamics was examined by Salam in 1951 [39]. We want to look for solutions other than the coupling constant expansion. At the time, it was very natural for us to put in a source for ϕ and order an iterative expansion by the number of derivatives with respect to the source. A variant of this method was used in my thesis to study the Bjorken model. However, we chose in GHK to keep things simple by writing down the equivalent leading order expansion in operator form. Note our approach is fully quantum mechanical and that the Goldstone theorem arguments depending on commutation relations are quantum mechanical as well. Here, the concept of a particle only has meaning in quantum mechanics and in no way is it necessary to calculate higher order corrections to have quantum results as has been argued elsewhere.

The leading approximation is obtained by replacing $i e_0 \phi^\mu \mathbf{q} \phi A_\mu$ in the Lagrangian by $\phi^\mu \eta A_\mu$. (The result is essentially the Boulware–Gilbert action with an extra scalar field). This “reduced Lagrangian” results in the linearized field equations:

$$\begin{aligned} F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu ; \\ \partial_\nu F^{\mu\nu} &= \phi^\mu \eta ; \\ \phi^\mu &= -\partial^\mu \phi - \eta A^\mu ; \\ \partial_\mu \phi^\mu &= 0 . \end{aligned}$$

These equations are soluble, since they are (rotated) free field equations. The diagonalized equations for the physical degrees of freedom are:

$$\begin{aligned} (-\partial^2 + \eta_1^2) \phi_1 &= 0 ; \\ -\partial^2 \phi_2 &= 0 ; \\ (-\partial^2 + \eta_1^2) A_k^T &= 0 . \end{aligned}$$

For convenience, we have made the assumption that η_1 carries the full value of the vacuum expectation of the scalar field (proportional to the expectation value of ϕ_2). The superscript T denotes the transverse part. The two components of A_k^T and the one component of ϕ_1 form the three physical components of a massive spin-one field while ϕ_2 is a spin-zero field. As previously mentioned, the Goldstone theorem is not valid, so there is no resulting massless particle associated with this theorem. If the Goldstone theorem were valid, ϕ_1 would be massless. It is very important to realize that it is an artifact of the lowest order approximation for the above action that ϕ_2 is massless. The excitation spectrum of this field is not constrained by any theorem. It is obvious to anyone who has calculated in QFT that there are higher order corrections and that, in fact, they diverge in 4 space-time dimensions and to get a final result that this theory need mass and coupling constant renormalization using experimental input. If this were not so, the theory would be basically inconsistent and free. This was obvious in 1964 and it is now. While knowledge beyond that which existed in 1964 is not necessary to make the preceding statements, if the reader wants to understand this in more detail than these obvious statements without calculating themselves I suggest that you refer to the famous paper by Coleman and E. Weinberg [40].

At this stage, it might be thought that we have written down an interesting, but possibly totally uncontrolled, approximation. There is no *a priori* reason to believe that this is even a meaningful approximation. The main result, that the massless spin-one field and the scalar field unite to form a spin-one massive excitation, could be negated by the next iteration of this approximation. However, this approximation meets an absolutely essential criterion that makes this unlikely. While the symmetry breaking removes full gauge invariance, current-conservation, which is the fundamental condition, is still respected. This is clear from the above linearized equations of motion. We can directly demonstrate that the mechanism, described earlier in this note for the failure of the Goldstone theorem, applies in this approximation.

The internal consistency and the consistency with exact results gives this approximation credence as a leading order of an actual solution. It is, in fact, not hard to make this the leading order of a well defined approximation scheme.

This solution of the action describes a 3-degree of freedom spin 1 particle and a 1-degree of freedom spin 0 particle. Goldstone's theorem does not apply, even though the current

$$J^\mu = i e_0 \phi^\mu \mathbf{q} \phi = \phi^\mu \cdot \eta$$

is conserved.

3. Summary and Comparisons

The GHK paper addresses two major issues in detail. As a general issue it explains in detail why gauge theories do not intrinsically require zero mass particles. This emphatically does not depend on a specific model, but is a consequence of the “leakage” of appropriate charges out of any surface. This is a fully quantum mechanical proof and it is exact, ultimately depending only on the general structural form of the radiation gauge propagator. As a specific issue the GHK paper examine a specific theory - Scalar Electrodynamics with the charge symmetry broken by requiring non-vanishing expectation value of the scalar field. We demonstrate that in a (non-coupling constant perturbative) self consistent leading order approximation that this theory has no Goldstone boson and that the original degrees of freedom combine to produce a massive vector particle with the mass depending on the symmetry breaking parameter and a single component real scalar particle with no intrinsic constraint on its mass.

There has been considerable recent discussion about the relative merit and content of the work of the three groups that first analyzed spontaneous symmetry breaking as now used in the unified theory of weak and electromagnetic interactions. Some of this discussion has been quite misleading and even wrong. The fact that the papers are over 45 years old means that for fair evaluation, it is important to be aware of both the sophistication and the limits of our understanding at that time. With this in mind, I will do my best to make clear some of the important differences between the three sets of work. An essential point to note is that understanding the possibility of “avoiding the Goldstone Theorem” (the motivation for all of these papers) encompasses two very distinct parts. The first part is to find a mechanism that makes it possible to spontaneously break symmetry in relativistic theories without producing massless particles. The second part is to find a specific example. It is possible to do either of these parts independently without an understanding of the other.

In regards to the mechanism, Brout and Englert make some comments, but give no quantitative argument. In the Higgs PRL paper, there is only the specific scalar electrodynamics example. In his physics letters paper Higgs observed that an argument given by Gilbert to negate the Goldstone theorem includes a term with a fixed vector such as can be found in radiation gauge electrodynamics. He makes no follow-up or extension of this argument in his PRL paper.

The GHK paper does an extensive analysis of the mechanism and shows that in broken gauge theories in the radiation gauge that charge leaks out of any volume and consequently is not conserved since currents continue to exist in the limit of volumes tending toward infinity. This negates the basic assumption of a conserved charge that is required by the Goldstone theorem. Since the radiation gauge removes all gauge modes, the Goldstone theorem, which applies in manifestly Lorentz invariant gauges, can only require massless gauge particles. This is discussed in considerable detail in our paper. This is an exact model independent result not limited to the explicit scalar field interaction and which, in particular, applies to composite models. When combined with the fully analyzed results of my previous related PRL paper it shows that there is no requirement that the photon have zero mass except when coupling constant perturbation theory is valid (this was probably first conjectured by Schwinger). A method for examining the photon mass through the Goldstone Theorem was introduced in my early PRL paper, used, but not discussed, to build the later GHK paper and analyzed in detail in my “Feldafing” talk given in the summer of 1965 [41]. I believe that it is correct to say that GHK was the only group to actually analyze the general mechanism. This observation is likely to be particularly important if the LHC does not reveal a fundamental scalar boson.

All three groups used the scalar electrodynamic Lagrangian to provide an example (in a first approximation) of a gauge theory with a broken symmetry that yields a massive spin one particle but their analysis is different.

EB does not fully construct the lowest order approximation. They do not show the “Higgs Boson” and while they do argue that their approximation respects current conservation it seems to me that their analysis is in general quite incomplete. They do make the essential identification of the massive parameter. They further “assume that the application of the theorem of Goldstone, Salam and Weinberg is straightforward and thus that the propagator of the field ϕ_2 which is “orthogonal” to ϕ_1 has a pole at $q=0$ which is not isolated.” It is true that in a covariant gauge that the theorem is valid and that there is a zero mass pole. That pole is not physical and is purely gauge, hence unphysical. It does not seem to me that is what EB are saying but I leave the interpretation to the reader.

The Higgs PRL is probably more complete than the EB paper although entirely classical. It fails to write down the full solution to the leading order equations displayed. He does have (the classical analogue) of a zero mass solution which is not mentioned or displayed. This solution is pure gauge but it needs to be noticed and properly handled. Higgs includes an explicit scalar self coupling in his model and writes down the equations for both scalar degrees of freedom. As mentioned above, GHK fully describes why the Goldstone boson is a gauge only excitation in manifestly covariant formulations and is not present in the physical radiation gauge. The results are exact and independent of the specific model analyzed in an approximation.

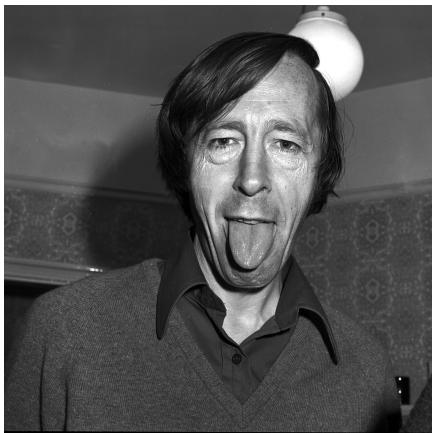
Recently it has been claimed that the GHK paper does not have the “Higgs boson”. This claim astonishes us. We, far more than any of the other groups, keep very careful track of the degrees of freedom of our scalar electrodynamics model. On the bottom of the right column of page 586 of the GHK paper are the three equations for the leading order approximations to the 4 physical degrees of freedom. We observe that the two degrees of freedom of the vector field combine with one scalar boson to form the three degrees of freedom of a massive spin one vector field. There is one remaining scalar field, ϕ_2 in our notation, which in our approximation has zero mass. That this mass is zero has absolutely nothing to do with any dynamical constraint including the Goldstone theorem. The Goldstone theorem, if valid here, would only constrain the mass of ϕ_1 . The zero mass is an artifact of how we pick the explicit action and the leading order approximation. This is different from the Higgs paper in that he puts in an explicit pure scalar interaction. In a 4 dimensional renormalizable theory that interaction is limited to being pure quartic. As was our practice mirroring that commonly used by Schwinger and associates, we did not put in this explicit quartic term in scalar electrodynamics but were fully aware that such a term is generated in higher approximations. Ultimately because, of renormalization, the GHK choice of the action is operationally identical to the one used by Higgs.

In summary, our purpose was to show that the Goldstone theorem did not constrain physical mass in scalar gauge theories. We demonstrated this generally and in a specific example. The mass of ϕ_2 happens to be zero in leading order, but as was obvious to us and every other experienced field theorist of that time, this would change order by order as the theory was iterated in a manner closely related to how it changes in unbroken scalar electromagnetism.



“Gang of five” Sakurai Prize 2010

The following show Tom Kibble and Gerald Guralnik hoping to obtain wisdom by emulation:





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